ANALYTICAL REPRESENTATION OF THE RESULTS OF A NUMERICAL DETERMINATION OF THE THERMODYNAMIC CHARACTERISTICS OF COMBUSTION PRODUCTS

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Approximate formulas are given for determining the ideal values of the specific impulses in a vacuum, the characteristic velocity, and the geometric expansion ratio of a supersonic nozzle.

Fuels whose combustion products are high-temperature, multicomponent, chemically reacting mixtures have found application in present-day heat technology. Results are published in the handbook [1] on the numerical integration of the problem of determining the ideal values of the thermodynamic characteristics of fuel combustion products, based on oxygen, fluorine, and other oxidants and fuels containing hydrogen, carbon, light metals, and other chemical elements. In the practical application of these data, it is necessary to resort to the interpolation of functions of two arguments — the stagnation flow pressure and the expansion ratio of the gas in the nozzle.

For the purpose of facilitating the problem of determining the optimum parameters of engines operating on the above-mentioned fuels, approximate analytical relations which describe the results of a numerical determination of the ideal values of the specific impulses in a vacuum, the characteristic velocity, and the geometric expansion ratio of the nozzle are obtained in the present paper. By specific impulse, we understand the ratio of the engine thrust to the mass fuel consumption; by the characteristic velocity we understand the product of the stagnation flow pressure and the area of the minimum cross section of the nozzle, referred to the mass fuel consumption; and by the geometric expansion ratio of the nozzle we understand the ratio of the area of the outlet section of the nozzle to the area of its minimum cross section.

The following quantities will be taken as the defining dimensionless criteria: the expansion ratio of the gas in the nozzle ε, equal to the ratio of the stagnation flow pressure to the static pressure in the outlet section of the nozzle; the relative pressure π, equal to the ratio of the stagnation flow pressure to the normal pressure $p_n = 101{,}325\ \mathrm{N/m^2}$.

For $\varepsilon = 50$-5000 and $\pi = 5$-500, the results of the calculations given in the handbook [1] can be described by the following empirical relations:

specific impulse in a vacuum,

$$I = I_0[1 + a_1\pi^{-0.5} + (a_2 + a_3\pi^{-0.25})f]^{-1},$$

where

$$f = \varepsilon^{-0.25} + a_4\varepsilon^{-0.75} - \varepsilon^{-1.25};$$

characteristic velocity,

$$\beta = \beta_0[1 - b_1\exp(-w_1\pi^{0.125}) + b_2\exp(-w_2\pi^{0.5})];$$

geometric expansion ratio of the nozzle,

$$F = c_1 + (c_2 + c_3\pi^{-c_4})\varepsilon^n,$$

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TABLE 1. Approximation Coefficients for an Oxygen–Hydrogen Fuel

Designation and dimension	Excess oxidant coefficient			
	0,4	0,6	0,8	1,0
I_0, m/sec	4756	4975	5091	5161
a_1	0	0	–0,015	0,020
a_2	0,402	0,512	0,626	0,805
a_3	0,002	0,046	0,179	0,220
a_4	1,83	1,36	0,89	0,67
β_0, m/sec	2439	2404	2359	2250
b_1	0,107	0,316	0,259	0,278
w_1	4,0	2,2	1,2	1,1
c_1	0,2	0,1	0	–0,1
c_2	0,375	0,369	0,328	0,378
c_3	0	0,040	0,113	–0,062
c_4	0	0,250	0,125	0,500
c_5	0,710	0,736	0,760	0,778
c_6	0	0	0	0,056

where

$$n = c_5 + c_6 \pi^{-0.25}.$$

Here I_0 is the specific impulse in a vacuum as ε and $\pi \to \infty$, and β_0 is the characteristic velocity as $\pi \to \infty$ for the assumed mathematical model of the problem. The other coefficients in these formulas are dimensionless quantities.

The numerical values of the quantities I_0 and β_0 and all the dimensionless coefficients occurring in the proposed relations have been determined for each specific fuel composition from the results of the numerical values of I, β, and F for a number of values of the defining criteria. As an example, Table 1 shows the results of a determination of these coefficients for the combustion products of a liquid oxygen–liquid hydrogen fuel. In the example considered, the quantities b_2 and w_2 are equal to zero.

The proposed relations have been verified according to the data published in [1] and show that in the range of variation of the criteria ε and π considered, the error in determining the quantities I and β does not exceed 0.1% and for the quantity F, 1%.

NOTATION

I — is the specific impulse in a vacuum;
β — is the characteristic velocity;
F — is the geometric expansion ratio;
ε — is the expansion ratio of gas in nozzle;
π — is the relative pressure;
p_n — is the normal pressure;
a, b, c, and w — are the dimensionless coefficients.

LITERATURE CITED

1. V. E. Alemasov et al., Thermodynamic and Thermophysical Properties of Combustion Products [in Russian], Vols. 1-7, Izd. VINITI, Akad. Nauk SSSR (1971-1974).